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**in a Model With Symmetry Group**  
**SUSY  $SO(10) \times \Delta(48) \times U(1)$**

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# Low Energy Phenomena in a Model With Symmetry Group

$$\text{SUSY } SO(10) \times \Delta(48) \times U(1)$$

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## Abstract

Fermion masses and mixing angles including that of neutrinos are studied in a model with symmetry group  $\text{SUSY } SO(10) \times \Delta(48) \times U(1)$ . Universality of Yukawa coupling of superfields is assumed. The resulting texture of mass matrices in the low energy region depends only on a single coupling constant and VEVs caused by necessary symmetry breaking. 13 parameters involving masses and mixing angles in the quark and charged lepton sector are successfully described by only five parameters with two of them determined by the scales of  $U(1)$ ,  $SO(10)$  and  $SU(5)$  symmetry breaking compatible with the requirement of grand unification and proton decay. The neutrino masses and mixing angles in the leptonic sector are also determined with the addition of a Majorana coupling term. It is found that LSND  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  events, atmospheric neutrino deficit and the mass limit put by hot dark matter can be naturally explained. Solar neutrino puzzle can be solved only by introducing

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sterile neutrino with one additional parameter. More precise measurements of  $\alpha_s(M_Z)$ ,  $V_{cb}$ ,  $V_{ub}/V_{cb}$ ,  $m_b$ ,  $m_t$ , as well as various CP violation and neutrino oscillation experiments will provide crucial tests of the present model.

## I. INTRODUCTION

The standard model (SM) is a great success. To understand the origin of the 18 free parameters(or 25 if neutrinos are massive) is a big challenge to high energy physics. Many efforts have been made along this direction. It was first observed by Gatto *et al*, Cabbibo and Maiani [1] that the Cabbibo angle is close to  $\sqrt{m_d/m_s}$ . This observation initiated the investigation of the texture structure with zero elements [2] in the fermion Yukawa coupling matrices. A general analysis and review of the previous studies on the texture structure was given by Raby in [3]. In [4,5] Anderson *et al.* presented an interesting model based on SUSY  $SO(10)$  and  $U(1)$  family symmetries with two zero textures '11' and '13' followed naturally. Though the texture '22' and '32' are not unique they could fit successfully the 13 observables in the quark and charged lepton sector with only six parameters [4].

In this paper we will follow their general considerations and make the following modifications:

1) We will use a discrete dihedral group  $\Delta(3n^2)$  with  $n = 4$ , a subgroup of  $SU(3)$ , as our family group instead of  $U(1)$  used in [5]. This kind of dihedral group was first used by Kaplan and Schmaltz [7] with  $n = 5$ . This group has only triplet and singlet irreducible representations, which is well suited for our purposes.

2) We will assume universality of Yukawa coupling before symmetry breaking so as to reduce the possible free parameters. In this kind of theories there are very rich structures above the GUT scale with many heavy fermions and scalars. All heavy fields must have some reasons to exist and interact which we do not understand at this moment. So we will just take the universality of coupling constants at the GUT scale as a working assumption and not worry about the possible radiative effects. If the phenomenology is all right, one has to be more serious to find a deeper reason for it.

3) We shall Choose some symmetry breaking directions different from [4,5] to ensure the needed Clebsch coefficients in order to eliminate further arbitrariness of the parameters.

Our paper is orginazied as follows: In section 2, we will present the results of the Yukawa

coupling matrices. The resulting masses and CKM quark mixings are also presented. In section 3, neutrino masses and CKM-type mixings in the lepton sector are presented. All existing neutrino experiments are discussed and found to be understandable in the present model. In section 4, the model with superfields and superpotential is explicitly presented. Conclusions and remarks are presented in the last section.

## II. YUKAWA COUPLING MATRICES

With the above considerations, a model based on group SUSY  $SO(10) \times \Delta(48) \times U(1)$  with a single coupling constant is constructed. Here  $U(1)$  is family-independent and introduced to distinguish various fields which belong to the same representations of  $SO(10) \times \Delta(48)$ . Yukawa coupling matrices which determine the masses and mixings of all quarks and charged leptons are obtained by carefully choosing the structure of the physical vacuum. We find

$$\Gamma_f^G = \frac{2}{3} \lambda_H \begin{pmatrix} 0 & \frac{3}{2} z_f \epsilon_P^2 & 0 \\ \frac{3}{2} z_f \epsilon_P^2 & 3 y_f \epsilon_G^2 e^{i\phi} & \frac{\sqrt{3}}{2} x_f \epsilon_G^2 \\ 0 & \frac{\sqrt{3}}{2} x_f \epsilon_G^2 & w_f \end{pmatrix} \quad (1)$$

for  $f = u, d, e$ , and

$$\Gamma_\nu^G = \frac{2}{3} \lambda_H \begin{pmatrix} 0 & \frac{3}{2} \frac{1}{|z_\nu|} & 0 \\ \frac{3}{2} \frac{1}{|z_\nu|} & 3 \frac{y_\nu}{z_\nu^2} \frac{\epsilon_G^2}{\epsilon_P^2} e^{i\phi} & \frac{\sqrt{3}}{2} \frac{x_\nu}{|z_\nu|} \frac{\epsilon_G^2}{\epsilon_P^2} \\ 0 & \frac{\sqrt{3}}{2} \frac{x_\nu}{|z_\nu|} \frac{\epsilon_G^2}{\epsilon_P^2} & w_f \end{pmatrix} \quad (2)$$

for Dirac type neutrino coupling.  $\lambda_H$  is the universal coupling constant expected to be of order one.  $\epsilon_G \equiv v_5/v_{10}$  and  $\epsilon_P \equiv v_5/\bar{M}_P$  with  $\bar{M}_P$ ,  $v_{10}$  and  $v_5$  being the VEVs for  $U(1)$ ,  $SO(10)$  and  $SU(5)$  symmetry breaking respectively.  $x_f$ ,  $y_f$ ,  $z_f$  and  $w_f$  ( $f = u, d, e, \nu$ ) are the Clebsch factors of  $SO(10)$  determined by the directions of symmetry breaking of the adjoints 45s. The following three directions have been chosen for symmetry breaking, namely:  $\langle A_X \rangle = v_{10} \text{diag.}(2, 2, 2, 2, 2) \otimes \tau_2$ ;  $\langle A_z \rangle = v_5 \text{diag.}(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, -2, -2) \otimes \tau_2$  and  $\langle A_u \rangle = v_5 \text{diag.}(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{1}{3}, \frac{1}{3}) \otimes \tau_2$ . The resulting Clebsch factors are:  $w_u = w_d =$

$w_e = w_\nu = 1$ ;  $x_u = -7/9$ ,  $x_d = -5/27$ ,  $x_e = 1$ ,  $x_\nu = -1/15$ ;  $y_u = 0$ ,  $y_d = y_e/3 = 2/27$ ,  $y_\nu = 4/45$ ;  $z_u = 1$ ,  $z_d = z_e = -27$ ,  $z_\nu = -15^3 = -3375$ .  $\phi$  is the physical CP phase<sup>1</sup> arising from the VEVs. The Clebsch factors associated with the symmetry breaking directions can be easily read off from effective operators which are obtained when the heavy fermion pairs are integrated out and decoupled

$$\begin{aligned}
W_{33} &= \left(\frac{2}{3}\lambda_H\right) \frac{1}{2} 16_3 10_1 16_3 \\
W_{32} &= \left(\frac{2}{3}\lambda_H\right) \frac{\sqrt{3}}{2} \epsilon_G^2 16_3 \left(\frac{A_z}{v_5}\right) \left(\frac{v_{10}}{A_X}\right) 10_1 \left(\frac{v_{10}}{A_X}\right) \left(\frac{A_z}{v_5}\right) \frac{1}{\sqrt{1 + \epsilon_P^2 \left(\frac{A_X}{v_{10}}\right)^6}} 16_2 \\
W_{22} &= \left(\frac{2}{3}\lambda_H\right) \frac{3}{2} \epsilon_G^2 16_2 \frac{1}{\sqrt{1 + \epsilon_P^2 \left(\frac{A_X}{v_{10}}\right)^6}} \left(\frac{A_u}{v_5}\right) \left(\frac{v_{10}}{A_X}\right) 10_1 \left(\frac{v_{10}}{A_X}\right) \left(\frac{A_u}{v_5}\right) \frac{1}{\sqrt{1 + \epsilon_P^2 \left(\frac{A_X}{v_{10}}\right)^6}} 16_2 \quad (3) \\
W_{12} &= \left(\frac{2}{3}\lambda_H\right) \frac{3}{2} \epsilon_P^2 16_1 \frac{1}{\sqrt{1 + \epsilon_P^2 \left(\frac{A_X}{v_{10}}\right)^6}} \left(\frac{A_X}{v_{10}}\right)^3 10_1 \left(\frac{A_X}{v_{10}}\right)^3 \frac{1}{\sqrt{1 + \epsilon_P^2 \left(\frac{A_X}{v_{10}}\right)^6}} 16_2
\end{aligned}$$

where the factor  $1/\sqrt{1 + \epsilon_P^2 \left(\frac{A_X}{v_{10}}\right)^6}$  arises from mixing. The  $\epsilon_P^2$  term in the square root is negligible for quarks and charged leptons, but it becomes dominant for the neutrinos due to the large Clebsch factor  $z_\nu$ . In obtaining the  $\Gamma_f^G$  matrices, some small terms arising from mixings between the chiral fermion  $16_i$  and heavy fermion pairs  $\psi_j(\bar{\psi}_j)$  are neglected. They are expected to change the numerical results no more than a few percent. The factor  $1/\sqrt{3}$  associated with the third family is due to the maximum mixing between the third family fermion and heavy fermions. This set of effective operators which lead to the above given Yukawa coupling matrices  $\Gamma_f^G$  is quite unique. Uniqueness of the structure of operator  $W_{12}$  was first observed by Anderson *et al* [4] from the mass ratios of  $m_e/m_\mu$  and  $m_d/m_s$ . The effective operator  $W_{33}$  is also fixed at the GUT scale [8,9,4] for the case of large  $\tan\beta$ . There is only one candidate of effective operator  $W_{22}$ , when the direction of breaking is chosen to be  $A_u$ , with Clebsch factors satisfying  $y_u : y_d : y_e = 0 : 1 : 3$  [10] so as to obtain a correct mass ratio  $m_\mu/m_s$ . The three parameters  $\lambda_H$ ,  $\epsilon_G$  and  $\epsilon_P$  are determined by the three measured mass ratios  $m_b/m_\tau$ ,  $m_\mu/m_\tau$  and  $m_e/m_\tau$ . Thus, the mass ratio  $m_c/m_t$  and the CKM mixing

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<sup>1</sup> We have rotated away other possible phases by a phase redefinition of the fermion fields.

elements  $V_{cb}$  and  $V_{ub}$  put strong constraint to an unique choice of the symmetry breaking direction  $A_z$  for effective operator  $W_{32}$ . Unlike many other models in which  $W_{33}$  is assumed to be a renormalizable interaction before symmetry breaking, the Yukawa couplings of all the quarks and leptons (both heavy and light) in the present model are generated at the GUT scale after the breakdown of the family group and  $SO(10)$ . Therefore, initial conditions of renormalization group (RG) evolution will be set at the GUT scale for all the quark and lepton Yukawa couplings. Consequently, one could avoid the possible Landau pole and flavor changing problems encountered in many other models due to RG running of the third family Yukawa couplings from the GUT scale to the Planck scale. The hierarchy among the three families is described by the two ratios  $\epsilon_G$  and  $\epsilon_P$ . Mass splittings between quarks and leptons as well as between the up and down quarks are determined by the Clebsch factors of  $SO(10)$ . From the GUT scale down to low energies, Renormalization Group (RG) evolution have been taken into account. Top-bottom splitting in the present model is mainly attributed to the hierarchy of the VEVs  $v_1$  and  $v_2$  of the two light Higgs doublets in the weak scale.

An adjoint 45  $A_X$  and a 16-D representation Higgs field  $\Phi$  ( $\bar{\Phi}$ ) are needed for breaking  $SO(10)$  down to  $SU(5)$ . Adjoint 45  $A_z$  and  $A_u$  are needed to break  $SU(5)$  further down to the standard model  $SU(3)_c \times SU_L \times U(1)_Y$ .

The numerical predictions for the quark, lepton masses and quark mixings are presented in table 1b with the input parameters and their values given in table 1a. RG effects have been considered following the standard scheme [8,4] by integrating the full two-loop RG equations from the GUT scale down to the weak scale using  $M_{SUSY} \simeq M_{WEAK} \simeq M_t \simeq 180\text{GeV}$ . From the weak scale down to the lower energy scale, three loops in QCD and two loops in QED are taken into consideration. SUSY threshold effects are not considered in detail here since the spectrum of sparticles is not yet determined. The bottom quark mass may receive corrections as large as 30% [9] due to large  $\tan\beta$ . However, it could be reduced by taking a suitable spectrum of superparticles. Therefore, one should not expect to have precise predictions until the spectrum of the sparticles is well determined. The strong coupling constant  $\alpha_s(M_Z)$  is taken to be a free parameter with values given by the present

experimental bounds  $\alpha_s(M_z) = 0.117 \pm 0.005$  [11] in the following.

**Table 1a.** Parameters and their values as a function of the strong coupling  $\alpha_s(M_Z)$  determined by  $m_b$ ,  $m_\tau$ ,  $m_\mu$ ,  $m_e$  and  $|V_{us}| = \lambda$ .

$\alpha_s(M_Z)$	$\phi$	$\epsilon_G \equiv v_5/v_{10}$	$\epsilon_P \equiv v_5/\bar{M}_P$	$\tan \beta$
0.110	73.4°	$2.66 \times 10^{-1}$	$0.89 \times 10^{-2}$	51
0.115	77.5°	$2.51 \times 10^{-1}$	$0.83 \times 10^{-2}$	55
0.120	81.5°	$2.34 \times 10^{-1}$	$0.77 \times 10^{-2}$	58

**Table 1b.** Observables and their predicted values with the values of the parameters given in the table 1a.

Input		Output with $\alpha_s(M_Z)$	0.110	0.115	0.120
$m_b(m_b)$ [GeV]	4.35	$m_t$ [GeV]	165	176	185
$m_\tau$ [GeV]	1.78	$m_c(m_c)$ [GeV]	1.14	1.30	1.37
$m_\mu$ [MeV]	105.6	$m_s(1\text{GeV})$ [MeV]	152	172	197
$m_e$ [MeV]	0.51	$m_d(1\text{GeV})$ [MeV]	6.5	7.2	8.0
$ V_{us}  \simeq \lambda$	0.22	$m_u(1\text{GeV})$ [MeV]	3.3	4.3	6.1
		$ V_{cb}  \simeq A\lambda^2$	0.045	0.045	0.043
		$ \frac{V_{ub}}{V_{cb}}  \simeq \lambda\sqrt{\rho^2 + \eta^2}$	0.053	0.056	0.063
		$ \frac{V_{td}}{V_{cb}}  \simeq \lambda\sqrt{(1-\rho)^2 + \eta^2}$	0.201	0.199	0.198

From table 1a, one sees that the model has large  $\tan \beta$  solution with  $\tan \beta \equiv v_2/v_1 \sim m_t/m_b$ . CP violation is near maximum with a phase  $\phi \sim 80^\circ$ . The vacuum structure between the GUT scale and Planck scale has a hierarchic structure  $\epsilon_G \equiv v_5/v_{10} \sim \lambda = 0.22$  and  $\epsilon_P \equiv v_5/\bar{M}_P \sim \lambda^3$ . Assuming  $(\bar{M}_P/M_P)^2 \simeq \alpha_G \simeq 1/24 \sim \lambda^2$  (here  $\alpha_G$  is the unified gauge coupling,  $M_P$  is the Planck mass), we have

$$\bar{M}_P = 2.5 \times 10^{18} \text{GeV},$$

$$v_{10} \simeq (0.86 \pm 0.16) \times 10^{17} \text{GeV}, \quad (4)$$



$$v_5 \equiv M_G \simeq (2.2 \pm 0.2) \times 10^{16} \text{GeV}$$

where the resulting value for the GUT scale agree well with the one obtained from the gauge coupling unification.  $\bar{M}_P$  is also very close to the reduced Planck scale  $\hat{M}_P = M_P/\sqrt{8\pi} = 2.4 \times 10^{18} \text{ GeV}$  and may be regarded as the scale for gravity unification.

It is seen from table 1b that the predictions on fermion masses and Cabbibo-Kobayashi-Maskawa (CKM) mixing angles fall in the range allowed by the experimental data [11–13]:

$$\begin{aligned} m_\tau &= 1777 \text{MeV} & m_\mu &= 105.6 \text{MeV} & m_e &= 0.51 \text{MeV} \\ m_b(m_b) &= (4.15 - 4.35) \text{GeV} & m_s(1 \text{GeV}) &= (105 - 230) \text{MeV} & m_d(1 \text{GeV}) &= (5.5 - 11.5) \text{MeV} \\ m_t(m_t) &= (157 - 191) \text{GeV} & m_c(m_c) &= (1.22 - 1.32) \text{GeV} & m_u(1 \text{GeV}) &= (3.1 - 6.4) \text{MeV} \end{aligned} \quad (5)$$

and

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9747 - 0.9759 & 0.218 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9738 - 0.9752 & 0.032 - 0.048 \\ 0.004 - 0.015 & 0.03 - 0.048 & 0.9988 - 0.9995 \end{pmatrix} \quad (6)$$

The model also gives a consistent prediction for the  $B^0$ - $\bar{B}^0$  mixing and CP violation in kaon decays (A detailed analysis will be presented elsewhere).

It is of interest to expand the above fermion Yukawa coupling matrices  $\Gamma_f^G$  in terms of the parameter  $\lambda = 0.22$  (the Cabbibo angle), which was found in [14] to be very useful for expanding the CKM mixing matrix. With the input values given in the table 1a, we find

$$\begin{aligned} \Gamma_u^G &\simeq \frac{2}{3} \lambda_H \begin{pmatrix} 0 & 0.97\lambda^6 & 0 \\ 0.97\lambda^6 & 0 & -0.89\lambda^2 \\ 0 & -0.89\lambda^2 & 1 \end{pmatrix}; \quad \Gamma_d^G \simeq \frac{2}{3} \lambda_H \begin{pmatrix} 0 & -1.27\lambda^4 & 0 \\ -1.27\lambda^4 & 1.39\lambda^3 e^{i0.86\pi/2} & -0.97\lambda^3 \\ 0 & -0.97\lambda^3 & 1 \end{pmatrix} \\ \Gamma_e^G &\simeq \frac{2}{3} \lambda_H \begin{pmatrix} 0 & -1.27\lambda^4 & 0 \\ -1.27\lambda^4 & 0.92\lambda^2 e^{i0.86\pi/2} & 1.16\lambda^2 \\ 0 & 1.16\lambda^2 & 1 \end{pmatrix}; \quad \Gamma_\nu^G \simeq \frac{2}{3} \lambda_H \begin{pmatrix} 0 & 0.86\lambda^5 & 0 \\ 0.86\lambda^5 & 0.85\lambda^7 e^{i0.86\pi/2} & -1.14\lambda^6 \\ 0 & -1.14\lambda^6 & 1 \end{pmatrix} \end{aligned} \quad (7)$$

for  $\alpha_s(M_Z) = 0.115$ .

### III. NEUTRINO MASSES AND MIXINGS

To find the neutrino masses and mixings will be crucial tests of the model. Many unification theories predict a see-saw type mass [15]  $m_{\nu_i} \sim m_{u_i}^2/M_N$  with  $u_i = u, c, t$  being up-type quarks. For  $M_N \simeq (10^{-3} \sim 10^{-4})M_{GUT} \simeq 10^{12} - 10^{13}$  GeV, one has

$$m_{\nu_e} < 10^{-7}eV, \quad m_{\nu_\mu} \sim 10^{-3}eV, \quad m_{\nu_\tau} \sim (3 - 21)eV \quad (8)$$

in this case solar neutrino anomalous could be explained by  $\nu_e \rightarrow \nu_\mu$  oscillation, and the mass of  $\nu_\tau$  is in the range relevant to hot dark matter. However, LSND events and atmospheric neutrino deficit can not be explained in this scenario.

By choosing Majorana type Yukawa coupling matrix differently, one can construct many models of neutrino mass matrix. We shall present one here, which is found to be of interest with the following texture:

$$M_N^G = \lambda_H v_{10} \frac{\epsilon_P^4}{\epsilon_G^2} \begin{pmatrix} 0 & 0 & \frac{1}{2}z_N \\ 0 & y_N & 0 \\ \frac{1}{2}z_N & 0 & w_N \end{pmatrix} \quad (9)$$

The corresponding effective operators are given by

$$\begin{aligned} W_{33}^N &= \lambda_H \frac{v_{10}}{2} \frac{\epsilon_P^4}{\epsilon_G^2} 16_3 \left( \frac{A_u}{v_5} \right) \left( \frac{\bar{\Phi}}{v_{10}/\sqrt{2}} \right) \left( \frac{\bar{\Phi}}{v_{10}/\sqrt{2}} \right) \left( \frac{A_{B-L}}{v_5} \right) 16_3 \\ W_{13}^N &= \lambda_H \frac{v_{10}}{2} \frac{\epsilon_P^4}{\epsilon_G^2} 16_1 \left( \frac{A_u}{v_5} \right) \left( \frac{\bar{\Phi}}{v_{10}/\sqrt{2}} \right) \left( \frac{\bar{\Phi}}{v_{10}/\sqrt{2}} \right) \left( \frac{A_{3R}}{v_5} \right) 16_3 \\ W_{22}^N &= \lambda_H \frac{v_{10}}{2} \frac{\epsilon_P^4}{\epsilon_G^2} 16_2 \left( \frac{A_u}{v_5} \right) \left( \frac{\bar{\Phi}}{v_{10}/\sqrt{2}} \right) \left( \frac{\bar{\Phi}}{v_{10}/\sqrt{2}} \right) \left( \frac{A_u}{v_5} \right) 16_2 \end{aligned} \quad (10)$$

where  $w_N$ ,  $y_N$  and  $z_N$  are Clebsch factors with  $w_N = 4/3$ ,  $y_N = 16/9$ ,  $z_N = 2/3$ . They are determined by additional 45s  $A_{B-L}$  and  $A_{3R}$  with  $\langle A_{B-L} \rangle = v_5 \text{diag.}(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0, 0) \otimes \tau_2$  and  $\langle A_{3R} \rangle = v_5 \text{diag.}(0, 0, 0, \frac{1}{2}, \frac{1}{2}) \otimes \tau_2$ . The 45  $A_{B-L}$  is also necessary for doublet-triplet mass splitting [16] in the Higgs  $10_1$ .

The light neutrino mass matrix is given via see-saw mechanism as follows

$$\begin{aligned}
M_\nu &= \Gamma_\nu^G (M_N^G)^{-1} (\Gamma_\nu^G)^\dagger v_2^2 = M_0 \begin{pmatrix} \frac{3}{4} \frac{z_N}{|z_\nu|} \frac{1}{y_N} & \frac{3}{2} \frac{z_N}{z_\nu^2} \frac{y_\nu}{y_N} \frac{\epsilon_G^2}{\epsilon_P^2} e^{-i\phi} & -\frac{\sqrt{3}}{4} \frac{z_N}{|z_\nu|} \frac{x_\nu}{y_N} \frac{\epsilon_G^2}{\epsilon_P^2} \\ \frac{3}{2} \frac{z_N}{z_\nu^2} \frac{y_\nu}{y_N} \frac{\epsilon_G^2}{\epsilon_P^2} e^{i\phi} & -3 \frac{w_N}{|z_\nu| z_N} - \sqrt{3} \frac{x_\nu}{|z_\nu|} \frac{\epsilon_G^2}{\epsilon_P^2} & 1 \\ -\frac{\sqrt{3}}{4} \frac{z_N}{|z_\nu|} \frac{x_\nu}{y_N} \frac{\epsilon_G^2}{\epsilon_P^2} & 1 & \frac{3}{2} \frac{z_N}{z_\nu^2} \frac{y_\nu}{y_N} \frac{\epsilon_G^2}{\epsilon_P^2} \end{pmatrix} \\
&= 2.1 \lambda_H \begin{pmatrix} 0.73\lambda^6 & 0.73\lambda^8 e^{-i0.86\pi/2} & -0.97\lambda^7 \\ 0.73\lambda^8 e^{i0.86\pi/2} & -0.86\lambda^4 & 1 \\ -0.97\lambda^7 & 1 & 0.73\lambda^8 \end{pmatrix} \quad (11)
\end{aligned}$$

with  $M_0 = \frac{2}{3} \frac{\epsilon_G^2}{\epsilon_P^4} \frac{1}{|z_\nu| z_N} \frac{1}{\eta_\nu} \frac{v_2}{v_{10}} v_2 \lambda_H$ . Here  $\eta_\nu$  is the RG evolution factor and estimated to be  $\eta_\nu \simeq 1.35$ . Diagonalizing the above mass matrix, we obtain masses of light Majorana neutrinos:

$$\begin{aligned}
\frac{m_{\nu_e}}{m_{\nu_\mu}} &= \frac{3}{4} \frac{z_N}{|z_\nu|} \frac{1}{y_N} = 0.83 \times 10^{-4}, \\
\frac{m_{\nu_\mu}}{m_{\nu_\tau}} &= 1 - 3 \frac{w_N}{|z_\nu| z_N} - \sqrt{3} \frac{x_\nu}{|z_\nu|} \frac{\epsilon_G^2}{\epsilon_P^2} \simeq 0.998 \\
m_{\nu_\tau} &\simeq M_0 \simeq 2.1 \lambda_H eV
\end{aligned} \quad (12)$$

The three heavy Majorana neutrinos have masses

$$\begin{aligned}
\frac{m_{N_1}}{m_{N_2}} &= \frac{z_N^2}{4w_N y_N} = 0.047, \quad \frac{m_{N_2}}{m_{N_3}} = \frac{y_N}{w_N} = 1.33 \\
m_{N_3} &= \frac{\epsilon_P^4}{\epsilon_G^2} w_N v_{10} \lambda_H \simeq 0.64 \times 10^{10} \lambda_H GeV
\end{aligned} \quad (13)$$

The CKM-type lepton mixing matrix is predicted to be

$$V_{LEP} = V_\nu V_e^\dagger = \begin{pmatrix} V_{\nu_e e} & V_{\nu_e \mu} & V_{\nu_e \tau} \\ V_{\nu_\mu e} & V_{\nu_\mu \mu} & V_{\nu_\mu \tau} \\ V_{\nu_\tau e} & V_{\nu_\tau \mu} & V_{\nu_\tau \tau} \end{pmatrix} = \begin{pmatrix} 0.9976 & 0.068 & 0.000 \\ -0.051 & 0.748 & 0.665 \\ 0.045 & -0.664 & 0.748 \end{pmatrix} \quad (14)$$

CP-violating effects are found to be small in the lepton mixing matrix. As a result we find the following:

1. a  $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$  short wave-length oscillation with

$$\Delta m_{e\mu}^2 = m_{\nu_\mu}^2 - m_{\nu_e}^2 \simeq (4-6)eV^2, \quad \sin^2 2\theta_{e\mu} \simeq 1.8 \times 10^{-2}, \quad (15)$$

which is consistent with the LSND experiment [17]

$$\Delta m_{e\mu}^2 = m_{\nu_\mu}^2 - m_{\nu_e}^2 \simeq (4 - 6)eV^2, \quad \sin^2 2\theta_{e\mu} \simeq 1.8 \times 10^{-2} \sim 3 \times 10^{-3}; \quad (16)$$

2. a  $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_\tau(\bar{\nu}_\tau)$  long-wave length oscillation with

$$\Delta m_{\mu\tau}^2 = m_{\nu_\tau}^2 - m_{\nu_\mu}^2 \simeq (1.6 - 2.4) \times 10^{-2} eV^2, \quad \sin^2 2\theta_{\mu\tau} \simeq 0.987, \quad (17)$$

which could explain the atmospheric neutrino deficit [18]:

$$\Delta m_{\mu\tau}^2 = m_{\nu_\tau}^2 - m_{\nu_\mu}^2 \simeq (0.5 - 2.4) \times 10^{-2} eV^2, \quad \sin^2 2\theta_{\mu\tau} \simeq 0.6 - 1.0, \quad (18)$$

with the best fit [18]

$$\Delta m_{\mu\tau}^2 = m_{\nu_\tau}^2 - m_{\nu_\mu}^2 \simeq 1.6 \times 10^{-2} eV^2, \quad \sin^2 2\theta_{\mu\tau} \simeq 1.0; \quad (19)$$

However,  $(\nu_\mu - \nu_\tau)$  oscillation will be beyond the reach of CHORUS/NOMAD and E803.

3. Two massive neutrinos  $\nu_\mu$  and  $\nu_\tau$  with

$$m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq (2.0 - 2.4)eV, \quad (20)$$

which fall in the range required by possible hot dark matter [19].

In this case, solar neutrino deficit has to be explained by oscillation between  $\nu_e$  and a sterile neutrino [20]  $\nu_s$ . Since strong bounds on the number of neutrino species both from the invisible  $Z^0$ -width and from primordial nucleosynthesis [21,22] require the additional neutrino to be sterile (singlet of  $SU(2) \times U(1)$ , or singlet of  $SO(10)$  in the GUT  $SO(10)$  model). Masses and mixings of the triplet sterile neutrinos can be chosen by introducing an additional singlet scalar with VEV  $v_s \simeq 450$  GeV. We find

$$\begin{aligned} m_{\nu_s} &= \lambda_H v_s^2 / v_{10} \simeq 2.4 \times 10^{-3} eV \\ \sin \theta_{es} &\simeq \frac{m_{\nu_L \nu_s}}{m_{\nu_s}} = \frac{v_2}{2v_s} \frac{\epsilon_P}{\epsilon_G^2} \simeq 3.8 \times 10^{-2} \end{aligned} \quad (21)$$

with the mixing angle consistent with the requirement necessary for primordial nucleosynthesis [23] given by [21]. The resulting parameters

$$\Delta m_{es}^2 = m_{\nu_s}^2 - m_{\nu_e}^2 \simeq 5.8 \times 10^{-6} eV^2, \quad \sin^2 2\theta_{es} \simeq 5.8 \times 10^{-3} \quad (22)$$

are consistent with the values [20] obtained from fitting the experimental data:

$$\Delta m_{es}^2 = m_{\nu_s}^2 - m_{\nu_e}^2 \simeq (4 - 9) \times 10^{-6} eV^2, \quad \sin^2 2\theta_{es} \simeq (1.6 - 14) \times 10^{-3} \quad (23)$$

This scenario can be tested by the next generation solar neutrino experiments in Sudbury Neutrino Observatory (SNO) and Super-kamiokanda (Super-K), both planning to start operation in 1996. From measuring neutral current events, one could identify  $\nu_e \rightarrow \nu_s$  or  $\nu_e \rightarrow \nu_\mu(\nu_\tau)$  since the sterile neutrinos have no weak gauge interactions. From measuring seasonal variation, one can further distinguish the small-angle MSW [24] oscillation from vacuum mixing oscillation.

#### IV. SUPERPOTENTIAL FOR FERMION YUKAWA INTERACTIONS

Non-Abelian discrete family symmetry  $\Delta(48)$  is important in the present model for constructing interesting texture structures of the Yukawa coupling matrices. It initiates from basic considerations that all three families are treated on the same footing at the GUT scale, namely the three families should belong to an irreducible triplet representation of a family symmetry group. Based on the well-known fact that masses of the three families have a hierarchical structure, the family symmetry group must be a group with at least rank three if the group is a continuous one. However, within the known simple continuous groups, it is difficult to find a rank three group which has irreducible triplet representations. This limitation of the continuous groups is thus avoided by their finite and disconnected subgroups. A simple example is the finite and disconnected group  $\Delta(48)$ , a subgroup of  $SU(3)$ .

The generators of the  $\Delta(3n^2)$  group consist of the matrices

$$E(0,0) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad (24)$$

and

$$A_n(p, q) = \begin{pmatrix} e^{i\frac{2\pi}{n}p} & 0 & 0 \\ 0 & e^{i\frac{2\pi}{n}q} & 0 \\ 0 & 0 & e^{-i\frac{2\pi}{n}(p+q)} \end{pmatrix} \quad (25)$$

It is clear that there are  $n^2$  different elements  $A_n(p, q)$  since if  $p$  is fixed,  $q$  can take on  $n$  different values. There are three different elements types,

$$A_n(p, q), \quad E_n(p, q) = A_n(p, q)E(0, 0), \quad C_n(p, q) = A_n(p, q)E^2(0, 0)$$

in the  $\Delta(3n^2)$  group, therefore the order of the  $\Delta(3n^2)$  group is  $3n^2$ . The irreducible representations of the  $\Delta(3n^2)$  groups consist of i)  $(n^2 - 1)/3$  triplets and three singlets when  $n/3$  is not an interger and ii)  $(n^2 - 3)/3$  triplets and nine singlets when  $n/3$  is an interger.

The characters of the triplet representations can be expressed [6]

$$\Delta_T^{m_1 m_2}(A_n(p, q)) = e^{i\frac{2\pi}{n}[m_1 p + m_2 q]} + e^{i\frac{2\pi}{n}[m_1 q - m_2(p+q)]} + e^{i\frac{2\pi}{n}[-m_1(p+q) + m_2 p]} \quad (26)$$

$$\Delta_T^{m_1 m_2}(E_n(p, q)) = \Delta_T^{m_1 m_2}(C_n(p, q)) = 0$$

with  $m_1, m_2 = 0, 1, \dots, n-1$ . Note that  $(-m_1 + m_2, -m_1)$  and  $(-m_2, m_1 - m_2)$  are equivalent to  $(m_1, m_2)$ .

One will see that  $\Delta(48)$  (i.e.,  $n=4$ ) is the smallest of the dihedral group  $\Delta(3n^2)$  with sufficient triplets for constructing interesting texture structures of the Yukawa coupling matrices.

The irreducible triplet representations of  $\Delta(48)$  consist of two complex triplets  $T_1(\bar{T}_1)$  and  $T_3(\bar{T}_3)$  and one real triplet  $T_2 = \bar{T}_2$  as well as three singlet representations. Their irreducible triplet representations can be expressed in terms of the matrix representation

$$\begin{aligned} T_1^{(1)} &= \text{diag.}(i, 1, -i), & T_1^{(2)} &= \text{diag.}(1, -i, i), & T_1^{(3)} &= \text{diag.}(-i, i, 1); \\ T_2^{(1)} &= \text{diag.}(-1, 1, -1), & T_2^{(2)} &= \text{diag.}(1, -1, -1), & T_2^{(3)} &= \text{diag.}(-1, -1, 1); \\ T_3^{(1)} &= \text{diag.}(i, -1, i), & T_3^{(2)} &= \text{diag.}(-1, i, i), & T_3^{(3)} &= \text{diag.}(i, i, -1) \end{aligned} \quad (27)$$

The matrix representations of  $\bar{T}_1^{(i)}$  and  $\bar{T}_3^{(i)}$  are the hermician conjugates of  $T_1^{(i)}$  and  $T_3^{(i)}$ . From this representation, we can explicitly construct the invariant tensors.

**Table 2**, Decomposition of the product of two triplets,  $T_i \otimes T_j$  and  $T_i \otimes \bar{T}_j$  in  $\Delta_{48}(SU(3))$ . Triplets  $T_i$  and  $\bar{T}_i$  are simply denoted by  $i$  and  $\bar{i}$  respectively. For example  $T_1 \otimes \bar{T}_1 = A \oplus T_3 \oplus \bar{T}_3 \equiv A3\bar{3}$ , here  $A$  represents a singlet.

$\Delta(48)$	1	$\bar{1}$	2	3	$\bar{3}$
1	$\bar{1}\bar{1}2$	$A3\bar{3}$	$\bar{1}3\bar{3}$	$123$	$12\bar{3}$
2	$\bar{1}3\bar{3}$	$13\bar{3}$	$A23$	$1\bar{1}\bar{3}$	$1\bar{1}3$
3	$123$	$\bar{1}23$	$1\bar{1}\bar{3}$	$2\bar{3}\bar{3}$	$A1\bar{1}$

All three families with  $3 \times 16 = 48$  chiral fermions are unified into a triplet 16-dimensional spinor representation of  $SO(10) \times \Delta(48)$ . Without losing generality, one can assign the three chiral families into the triplet representation  $T_1$ , which may be simply denoted as  $\hat{16} = 16_i T_1^{(i)}$ . All the fermions are assumed to obtain their masses through a single  $10_1$  of  $SO(10)$  into which the needed two Higgs doublets are unified. The model could allow a triplet sterile neutrino with small mixings with the ordinary neutrinos. A singlet scalar near the electroweak scale is necessary to generate small masses for the sterile neutrinos.

Superpotentials which lead to the above texture structures (eqs. (1), (2) and (9)) with zeros and effective operators (eqs. (3) and (10)) are found to be

$$\begin{aligned}
W_Y = & \sum_{a=0}^3 \psi_{a1} 10_1 \psi_{a2} + \bar{\psi}_{22} \chi \psi_{13} + \bar{\psi}_{21} \chi_2 \psi_{13} + \bar{\psi}_{32} \chi \psi_{23} + \bar{\psi}_{31} \chi_3 \psi_{23} \\
& + \bar{\psi}_{02} \chi \psi_{33} + \bar{\psi}_{01} \chi_0 \psi_{33} + \bar{\psi}_{33} A_X \psi_3 + \bar{\psi}_3 A_X \psi_2 + \bar{\psi}_2 A_X \psi_1 \\
& + (\bar{\psi}_{11} \chi_1 + \bar{\psi}_{12} \chi + \bar{\psi}_{13} A_z + \bar{\psi}_{23} A_u + \bar{\psi}_1 Y) \hat{16} \\
& + \sum_{a=0}^3 \sum_{j=1}^3 S_G \bar{\psi}_{aj} \psi_{aj} + \sum_{i=1}^2 (\bar{\psi}_{i3} A_X \psi_{i3} + S_I \bar{\psi}_i \psi_i) + S_I \bar{\psi}_{33} \psi_{33} + S_P \bar{\psi}_3 \psi_3
\end{aligned} \tag{28}$$

for the fermion Yukawa coupling matrices, and

$$\begin{aligned}
W_R = & \sum_{i=1}^3 (\psi'_{i1} 10_3 \psi'_{i2} + \bar{\psi}'_{i1} \chi'_i \psi + \bar{\psi}'_{i2} \chi' \psi'_{i3} + \bar{\psi}'_{i1} A_i \psi') + (\bar{\psi}' X + \bar{\psi} A_u) \hat{16} + \bar{\Phi} 10_3 \bar{\Phi} \\
& + \sum_{i=1}^3 \sum_{j=1}^2 S_G \bar{\psi}'_{ij} \psi'_{ij} + S_P (\sum_{i=1}^3 \bar{\psi}'_{i3} \psi'_{i3} + \bar{\psi} \psi + \bar{\psi}' \psi')
\end{aligned}$$

for the right-handed Majorana neutrinos, and

$$W_S = \bar{\psi}'_1 10_1 \psi'_2 + \bar{\psi}'_1 \Phi \nu_s + \bar{\psi}'_2 \phi_s \hat{16} + (\bar{\nu}_s \phi_s N_s + h.c.) + S_I \bar{N}_s N_s \quad (29)$$

for the sterile neutrino masses and their mixings with the ordinary neutrinos.

In the above superpotentials, each term is ensured by the  $U(1)$  symmetry. An appropriate assignment of  $U(1)$  charges for the various fields is implied. All  $\psi$  fields are triplet 16-D spinor heavy fermions. Where the fields  $\psi_{i3}\{\bar{\psi}_{i3}\}$ ,  $\psi'_{i3}\{\bar{\psi}'_{i3}\}$ ,  $\psi_i\{\bar{\psi}_i\}$  ( $i = 1, 2, 3$ ),  $\psi'_1\{\bar{\psi}'_1\}$ ,  $\psi'_2\{\bar{\psi}'_2\}$ ,  $\psi\{\bar{\psi}\}$  and  $\psi'\{\bar{\psi}'\}$  belong to  $(16, T_1)\{(\bar{16}, \bar{T}_1)\}$  representations of  $SO(10) \times \Delta_{48}(SU(3))$ ;  $\psi_{11}\{\bar{\psi}_{11}\}$  and  $\psi_{12}\{\bar{\psi}_{12}\}$  belong to  $(16, T_2)\{(\bar{16}, \bar{T}_2)\}$ ;  $\psi_{i1}\{\bar{\psi}_{i1}\}$  and  $\bar{\psi}_{i2}\{\psi_{i2}\}$  ( $i = 2, 3, 0$ ) belong to  $(16, T_3)\{(\bar{16}, \bar{T}_3)\}$ ;  $\psi'_{i1}\{\bar{\psi}'_{i1}\}$  and  $\bar{\psi}'_{i2}\{\psi'_{i2}\}$  ( $i = 1, 2$ ) belong to  $(16, \bar{T}_3)\{(\bar{16}, T_3)\}$ ;  $\psi'_{31}\{\bar{\psi}'_{31}\}$  and  $\psi'_{32}\{\bar{\psi}'_{32}\}$  belong to  $(16, T_2)\{(\bar{16}, \bar{T}_2)\}$ ;  $X$ ,  $Y$ ,  $S_I$ ,  $S_P$  and  $\phi_s$  are singlets of  $SO(10) \times \Delta_{48}(SU(3))$ .  $\nu_s$  and  $N_s$  are  $SO(10)$  singlet and  $\Delta(48)$  triplet fermions.  $10_3$  is an additional  $SO(10)$  10-representation heavy scalar. All  $SO(10)$  singlet  $\chi$  fields are triplets of  $\Delta(48)$ . Where  $(\chi_1, \chi_2, \chi_3, \chi_0, \chi)$  belong to triplet representations  $(\bar{T}_3, T_3, \bar{T}_1, T_2, \bar{T}_3)$  respectively;  $(\chi'_1, \chi'_2, \chi'_3, \chi')$  belong to triplet representations  $(\bar{T}_1, T_2, T_3, T_3)$  respectively. With the above assignment for various fields, one can check that once the triplet field  $\chi$  develops VEV only along the third direction, i.e.,  $\langle \chi^{(3)} \rangle \neq 0$ , and  $\chi'$  develops VEV only along the second direction, i.e.,  $\langle \chi'^{(2)} \rangle \neq 0$ , the resulting fermion Yukawa coupling matrices at the GUT scale will be automatically forced, due to the special features of  $\Delta(48)$ , into an interesting texture structure with four non-zero textures '33', '32', '22' and '12' which are characterized by  $\chi_1$ ,  $\chi_2$ ,  $\chi_3$ , and  $\chi_0$  respectively, and the resulting right-handed Majorana neutrino mass matrix is forced into three non-zero textures '33', '13' and '22' which are characterized by  $\chi'_1$ ,  $\chi'_2$ , and  $\chi'_3$  respectively. It is seen that five triplets are needed. Where one triplet is necessary for unity of the three family fermions, and four triplets are required for obtaining the needed minimal non-zero textures.

The symmetry breaking scenario and the structure of the physical vacuum are considered as follows

$$\begin{aligned} SO(10) \times \Delta(48) \times U(1) &\xrightarrow{\bar{M}_P} SO(10) \times \Delta(48) \xrightarrow{v_{10}} SU(5) \times \Delta(48) \\ &\xrightarrow{v_5} SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{v_1, v_2} SU(3)_c \times U(1)_{em} \end{aligned} \quad (30)$$



and:  $\langle S_P \rangle = \bar{M}_P$ ,  $\langle X \rangle = v_{10} = \langle S_I \rangle$ ,  $\langle \Phi^{(16)} \rangle = \langle \bar{\Phi}^{(16)} \rangle = v_{10}/\sqrt{2}$ ,  $\langle Y \rangle = v_5 = \langle S_G \rangle$ ,  $\langle \chi^{(3)} \rangle = \langle \chi_a^{(i)} \rangle = \langle \chi'^{(2)} \rangle = \langle \chi_j'^{(i)} \rangle = v_5$  with  $(i = 1, 2, 3; a = 0, 1, 2, 3; j = 1, 2, 3)$ ,  $\langle \chi^{(1)} \rangle = \langle \chi^{(2)} \rangle = \langle \chi'^{(1)} \rangle = \langle \chi'^{(3)} \rangle = 0$ ,  $\langle \phi_s \rangle = v_s \simeq 450$  GeV,  $\langle H_2 \rangle = v_2 = v \sin \beta$  with  $v = \sqrt{v_1^2 + v_2^2} = 246$  GeV.

## V. CONCLUSIONS

It is amazing that nature has allowed us to make predictions in terms of a single Yukawa coupling constant and a set of VEVs determined by the structure of the vacuum and to understand the low energy physics from the Planck scale physics. The present model has provided a consistent picture on the 28 parameters in SM model with massive neutrinos. The neutrino sector is of special interest to further study. Though the recent LSND experiment, atmospheric neutrino deficit, and hot dark matter could be simultaneously explained in the present model, however, solar neutrino puzzle can be understood by introducing an SO(10) singlet sterile neutrino. It is expected that more precise measurements from various low energy experiments in the near future could provide crucial tests on the present model.

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